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Generalization of Weak Contraction Mapping In Modular Space For Some Fixed Point Results in Integral Type

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ABSTRACT: In this paper we introduced the new results for a generalised weak contractive mapping in modular metric space using integral type.

I. INTRODUCTION AND PRELIMINARIES

Weak contraction in Hilbert space is firstly introduced by Alber and Guerre- Delabriere in 1997, and then later it can be proved by Rhoades on the basis of complete metric spaces. It defines that If (X, d) is a metric space and T is a mapping from X to X then T is said to be weakly contractive, If

$$d\big(T(x),\ T(y)\big) \le d(x,y) - \psi\, d(x,y) \qquad \ldots(1)$$

When $\psi : [0,\infty) \to [0,\infty)$ is a non decreasing continuous function such that $\psi(u) = 0$ $iff\ u = 0$ Also the concept is re-generalized by the author's Dutta and Chaudhary for the generalization of contraction in metric space.

The author Nakano first introduced the conceptual theory of modular metric space and further generalization can be done by Musielak and Orlicz. Now the theory Modular metric space is mostly used the study of orlicz spaces whose applications to integral operators, approximations and fixed point theory.

Definition1.1. Let X be an arbitrary vector space over$K = (\mathbb{R}\ or\ \mathbb{C})$.

A) A functional $\rho: X \to [0,\infty]$ is called modular if:

(i) $\rho(x) = 0\ iff\ x = 0.$

(ii) $\rho(\alpha x) = \rho(x)\ for\ \alpha \in K\ with\ |\alpha| = 1,\ for\ all\ x \in X.$

(iii) $\rho(\alpha x + \beta y) \le \rho(x) + \rho(y)\ if\ \alpha,\beta \ge 0, \alpha+\beta = 1, for\ all\ x, y\ \in X$

If (iii) is replaced by

(iv) $\rho(\alpha x + \beta y) \le \alpha\rho(x) + \beta\rho(y)\ if\ \alpha,\beta \ge 0, \alpha+\beta = 1, for\ all\ x, y\ \in X$

Then the modular ρ is called convex modular.

B) A modular ρ defines a corresponding modular space; i.e. the spaceX_ρ given by:

$$X_\rho = \{x \in X;\ \rho(\alpha x) \to 0\ as\ \alpha \to 0\} \qquad \ldots(2)$$

Definition1.2. Let X_p be a modular space.

a) A sequence $(x_n)_{n\in N}$ in X_ρ is said to be:

i) ρ-convergent to x If $\rho(x_n - x) \to 0\ as\ n \to \infty$.

ii) ρ-Cauchy if $\rho(x_n - x_m) \to 0\ as\ n \to \infty$.

b) $X_\rho$isρ-Complete if every ρ-Cauchy sequence is ρ-convergent.

c) A subset B $\subset X_\rho$is said to be ρ-closed if for any sequence $(x_n)_{n\in N} \subset Band\ \ x_n \to x$ we have $x \in$ B

d) A subset B $\subset X_\rho$ is said to be ρ-bounded if $\delta_\rho(B) = \sup\rho(x-y) < \infty$

for all $x, y \in B$, where $\delta_\rho(B)$ is the ρ-diameter of B.

e) ρhas a Fatou property if

$\rho(x-y) \le \liminf \rho(x_n - y_n)$.

Whenever $x_n \to x$ and $y_n \to y\ as\ n \to \infty$.

f) ρ is said to satisfy the Δ_2 –condition if $\rho(2x_n) \to 0,\ whenever\ \rho(x_n) \to 0$

$as\ n \to \infty.$ $\qquad \ldots(3)$

Definition 1.3. Let X be a nonempty set and F: X → X. A point $x \in X$ is a fixed point of F iff$F\ x = x$.

Definition 1.4. Let Q be a subset of real number $\mathbb{R}$. A mapping $F: Q \to \mathbb{R}$ is called monotone increasing (or monotone non-decreasing) $x \le y$ iff $T(x) \le T(y)$, for all x and y are elements in Q. A mapping $F: Q \to \mathbb{R}$ is called monotone decreasing (or monotone non-increasing), $x \ge y$ Iff $T(x) \ge T(y)$ for all x and y are elements in Q.

II. MAIN SECTION

2. A Fixed point Theorem of Integral Type for a generalized weak contractionin modular spaces

Proposition 2.1. Let ρbea modular space on X, If$\alpha, \beta \in R^+$ with $\beta \ge \alpha$, then $\rho(\alpha x) \le \rho(\beta x)$

Proof: In case $\alpha = \beta$, we are done, suppose $\beta > \alpha$, and then one has $\frac{\alpha}{\beta} < 1$ and then $\rho(\alpha x) = \rho\left(\frac{\alpha}{\beta}\beta x\right)$

$$= \rho\left(\frac{\alpha}{\beta}\beta x + \left(1 - \frac{\alpha}{\beta}\right)(0)\right) \quad \ldots(4)$$

$$\le \rho(\beta x) + \rho(0)$$
$$= \rho(\beta x)$$

Proposition 2.2. Let X_ρ be a modular space inρ satisfy the Δ_2-condition and Let $\{x_n\}$ n N be a sequence in X_ρ. If $\rho(c\,(x_n - x_{n-1})) \to 0$ as n$\to\infty$, then $\rho(al\,(x_n - x_{n-1})) \to 0$as n$\to\infty$, where c, l, a R^+ $with\, c > l\, and\, \frac{l}{c} + \frac{1}{a} = 1$.

Proof: Since $\rho(c\,(x_n - x_{n-1})) \to 0$ as n$\to\infty$, then by Δ_2-condition we get

$$\rho(2^m . c\,(x_n - x_{n-1})) \to 0 \text{as n} \to \infty \quad \ldots(5)$$

Now from proposition for m $\in N$ using Sandwich theorem, we have

$$\rho(2N\,c\,(x_n - x_{n-1})) \to 0 \text{as n} \to \infty, \quad \ldots(6)$$

for n N we have $\frac{l}{c} + \frac{1}{a} = 1$, we get

$$al = (a-1)c \ge c$$

Then there exist $N_a \in \mathbb{N}$ such that

$$2\,(N_a - 1)c \le (a-1)c \le 2\,(Na)c \quad \ldots(7)$$

By Proposition we get

$$\rho(2(N_a - 1)c\,(x_n - x_{n-1}))$$
$$\le \rho((a-1)c\,(x_n - x_{n-1})) \quad \ldots(8)$$
$$\le \rho(2(N_a)c\,(x_n - x_{n-1}))$$

From (6) and (8), we obtain

$$= \lim_{n\to\infty} \rho(\alpha\, l\,(x_n - x_{n-1}))$$
$$= \lim_{n\to\infty} \rho((\alpha - 1)\ (x_n - x_{n-1})) = 0 \quad \ldots(9)$$

Theorem 2.3. Let X_ρ be a ρ-complete modular space, whereρ satisfy the Δ_2-condition and Let c, l, R^+ $with\, c > l\, and R : X_\rho \to X_\rho$ be a mapping satisfying the inequality

$$\int_0^{\psi(\rho(c(R_x - R_y)))} \xi(u)du \le \int_0^{\psi(\rho(l(x-y))) - \phi(\rho(l(x-y)))} \xi(u)du \quad \ldots(10)$$

For all x, y X_ρ, where $\psi, \phi : [0,1) \to [0,1)$ are both continuous and monotonic non decreasing function with $\psi(u) = \phi(u) = 0$ Iff $u = 0$, $\xi : R_+ \to R_+$ is a Lebesgue integrable mapping is summable on each compact subset of [0,1), nonnegative and $\int_0^{\epsilon} \xi(u)du = 0$, $\forall\ \ \varepsilon > 0$. Then R is a unique fixed point

Proof: Let the sequence be $\{x_n\}$n$\in \mathbb{N}$ $and\, x_0 \in X_\rho\ by\, x_n = Rx_{n-1}, n = 1, 2, 3, \ldots$

We firstly proves the sequence $\{\rho(c(Rx_n - Rx_{n-1}))\} \to 0$perhaps

$$\int_0^{\psi(\rho(c(x_n - x_{n+1})))} \xi(u)du \le \int_0^{\psi(\rho(l(x_{n-1} - x_n))) - \phi(\rho(l(x_{n-1} - x_n)))} \xi(u)du$$
$$\le \int_0^{\psi(\rho(l(x_{n-1} - x_n)))} \xi(u)du \quad \ldots(11)$$

By monotonic non decreasing function of ψ and proposition, we have

$$\int_0^{(\rho(c(x_n - x_{n+1})))} \xi(u)du \le \int_0^{(\rho(l(x_{n-1} - x_n)))} \xi(u)du$$
$$\le \int_0^{(\rho(c(x_{n-1} - x_n)))} \xi(u)du \quad \ldots(12)$$

Therefore the sequence $[\rho(c(x_{n-1} - x_n))]$ is monotone decreasing and bounded below, Hence there exists $r \ge 0$ such that

$$\lim_{n\to\infty} \rho(c\,(x_n - x_{n-1})) = s \quad \ldots(13)$$

If $s > 0, taking\ n \to \infty$ in the inequality (11), we get

$$\psi(s) \le \psi(s) - \phi(s)$$
$$< \psi(s) \qquad \ldots(14)$$

Which is a contraction, thus $s = 0$ we have

$$\lim_{n\to\infty} \rho(c\ (x_n - x_{n-1})) \to 0 \qquad \ldots(15)$$

Next, Now we prove the sequence $\{cx_n\}$n$\in \mathbb{N}$ is ρ-cauchy. Suppose that $\{cx_n\}$n$\in \mathbb{N}$ is not ρ-cauchy, then $\exists\ \varepsilon > 0$ and subsequence $\{x_{pk}\}$ and $\{x_{qk}\}$ $with\ pk > qk \ge k$ such that

$$\rho\big(c\ (x_{pk} - x_{qk})\big) \ge \varepsilon$$
$$\rho\big(c\ (x_{pk-1} - x_{qk})\big) < \varepsilon \qquad \ldots(16)$$

Now, let $\beta \in R^+$ such that $\frac{l}{c} + \frac{1}{\beta} = 1$, then we get

$$\int_0^{\psi\left(\rho\left(c(x_{pk}-x_{qk})\right)\right)} \xi(u)du \le \int_0^{\psi\left(\rho\left(l(x_{pk-1}-x_{qk-1})\right)\right)-\phi\left(\rho\left(l(x_{pk-1}-x_{qk-1})\right)\right)} \xi(u)du$$
$$\le \int_0^{\psi\left(\rho\left(l(x_{pk-1}-x_{qk-1})\right)\right)} \xi(u)du \qquad \ldots(17)$$
$$\int_0^{\psi\left(\rho\left(c(x_{pk}-x_{qk})\right)\right)} \xi(u)du \le \int_0^{\psi\left(\rho\left(l(x_{pk-1}-x_{qk-1})\right)\right)} \xi(u)du$$

Which implies that $\rho\left(c(x_{pk} - x_{qk})\right) \le \rho\left(l(x_{pk-1} - x_{qk-1})\right)$ $\qquad \ldots(18)$

we have

$$\int_0^{\left(\rho\left(l(x_{pk-1}-x_{qk-1})\right)\right)} \xi(u)du = \int_0^{\left(\rho\left(l(x_{pk-1}-x_{qk}+x_{qk}-x_{qk-1})\right)\right)} \xi(u)du$$
$$= \int_0^{\rho\left(\frac{1}{c}c(x_{pk-1}-x_{qk})+\frac{1}{\beta}\beta l(x_{qk}-x_{qk-1})\right)} \xi(u)du \qquad \ldots(19)$$
$$\le \int_0^{\rho\left(\frac{1}{c}c(x_{pk-1}-x_{qk})\right)+\rho\left(\frac{1}{\beta}\beta l(x_{qk}-x_{qk-1})\right)} \xi(u)du$$
$$< \int_0^{\varepsilon + \rho\left(\beta l(x_{qk}-x_{qk-1})\right)} \xi(u)du$$

From (16), (18) and (19), we get

$$\varepsilon \le \rho\left(c(x_{pk} - x_{qk})\right)$$
$$\le \rho\left(l(x_{pk-1} - x_{qk-1})\right)$$
$$< \varepsilon + \rho\left(\alpha l(x_{qk} - x_{qk-1})\right) \qquad \ldots(20)$$

From (15) and proposition [2.2] we have

$$\lim_{k\to\infty} \rho\left(\beta l(x_{pk} - x_{qk-1})\right) = 0 \qquad \ldots(21)$$

From (20) and (21) we obtain

$$\rho\left(c(x_{pk} - x_{qk})\right) = \lim_{k\to\infty} \rho\left(l(x_{pk-1} - x_{qk-1})\right) = \epsilon \qquad \ldots(22)$$

$$\lim_{k\to\infty} \int_0^{\left(\rho\left(l(x_{pk-1}-x_{qk-1})\right)\right)} \xi(u)du = \epsilon$$

Letting $k \to \infty$ in (17), by prop of ψ and Eq. (22) we get

$$\psi(\epsilon) \le \psi(\epsilon) - \psi(\epsilon) < \psi(\epsilon) \qquad \ldots(23)$$

which is contradiction. Therefore $\{cx_n\}$n$\in \mathbb{N}$ is ρ-cauchy. Since X_ρ is ρ-complete there exist a point t X_ρ such that $\lim_{n\to\infty} \rho\big(c(x_t - t)\big) \to 0$, Next we prove that t is a unique fixed point of T.

putting $x = x_{p-1}\ and\ y = t\ in$ (10), we obtain

$$\int_0^{\psi\left(\rho\left(c(x_p-Rt)\right)\right)} \xi(u)du \le \int_0^{\psi\left(\rho\left(l(x_{p-1}-t)\right)\right)-\phi\left(\rho\left(l(x_{p-1}-t)\right)\right)} \xi(u)du \qquad \ldots(24)$$

Now at $n \to \infty$, in the inequality we have

$$\psi\left(\rho\big(c(t - Rt)\big)\right) \le \psi(0) - \phi(0) = 0 \qquad \ldots(25)$$

$\rho\big(c(Rt - t)\big) = 0\ and\ Rt = t$, Suppose that there exist $s \in X_\rho$ such that $Rs = s\ and\ s \ne t$ therefore we have

$$\int_0^{\psi(\rho(c(t-s)))} \xi(u)du \leq \int_0^{\psi(\rho(c(Rt-Rs)))} \xi(u)du$$

$$\leq \int_0^{\psi(\rho(l(t-s)))-\phi(\rho(l(t-s)))} \xi(u)du \qquad \ldots(26)$$

$$\leq \int_0^{\psi(\rho(l(t-s)))} \xi(u)du$$

$$\leq \int_0^{\psi(\rho(c(t-s)))} \xi(u)du$$

Which is a contraction. Hence $t = s$. Therefore R is a unique fixed point.

Corollary 2.4: Let X_ρ be a ρ-Complete modular space where ρ satisfy the Δ_2-condition and Let c, l, □ R^+ *with* $c > l$ *and* $R : X_\rho \to X_\rho$ be a mapping satisfying the inequality

$$\int_0^{(\rho(c(R_x - R_y)))} \xi(u)du \leq \int_0^{(\rho(l(x-y)))-\phi(\rho(l(x-y)))} \xi(u)du \qquad \ldots(27)$$

For all x, y □ X_ρ, where $\phi : [0,1) \to [0,1)$ are both continuous and monotonic non decreasing function with $\phi(u) = 0$ Iff $u = 0$, $\xi : R_+ \to R_+$ is a Lebesgue integrable mapping is summable on each compact subset of [0,1), nonnegative and $\int_0^{\epsilon} \xi(u)du = 0$, ∀ $\varepsilon > 0$. Then R has a unique fixed point.

Proof: Take $\psi(u) = u$ we obtain this corollary.

Theorem 2.5: Let X_ρ be a ρ-Complete modular space where ρ satisfy the Δ_2-condition and Let c, l, □ R^+ *with* $c > l$ *and* $R : X_\rho \to X_\rho$ be a mapping satisfying the inequality

$$\int_0^{\psi(\rho(R_x - R_y))} \xi(u)du \leq \int_0^{\psi(m(x,y))-\phi(m(x,y))} \xi(u)du \ldots(28)$$

For all x, y □ X_ρ, where

$$m(x,y) = max\left\{\begin{matrix} \rho(x-y), \rho(x-Rx), \rho(y-Ry), \\ \left\{\rho\left(\frac{1}{2}(x-Ry)\right) + \rho\left(\frac{1}{2}(y-Rx)\right)\frac{1}{2}\right\} \end{matrix}\right\} \qquad \ldots(29)$$

and ψ, $\phi : [0,\infty) \to [0,\infty)$ are both continuous and monotonic non decreasing function with $\phi(u) = \phi(u) = 0$ Iff $u = 0$, Then $\xi : R_+ \to R_+$ is a Lebesgue integrable mapping is summable on each compact subset of $[0,\infty)$ nonnegative and $\int_0^{\epsilon} \xi(u)du = 0$, ∀ $\varepsilon > 0$. Then R has a unique fixed point.

Proof: Firstly we prove that the sequence $\{\psi(c(R^q x - R^{q-1}x))\} \to 0$, since

$$\int_0^{\psi(\rho(R^q x - R^{q-1}x))} \xi(u)du \leq \int_0^{\psi(m(R^{q-1}x - R^{q-2}x))-\phi(m(R^{q-1}x - R^{q-2}x))} \xi(u)du \qquad \ldots(30)$$

$$\leq \int_0^{\psi(m(R^{q-1}x - R^{q-2}x))} \xi(u)du$$

Monotone non-decreasing of ψ we have

$$\rho(R^q x - R^{q-1}x) \leq m(R^{q-1}x - R^{q-2}x) \qquad \ldots(31)$$

By definition of $m(x,y)$, we get

$$\int_0^{\psi(m(R^{q-1}x - R^{q-2}x))} \xi(u)du = \int_0^{\psi \max\left\{\begin{matrix}\rho(R^{q-1}x - R^{q-2}x), \rho(R^q x - R^{q-1}x), \\ \left[\frac{\rho(1/2)(R^q x - R^{q-2}x)}{2}\right]\end{matrix}\right\}} \xi(u)du \qquad \ldots(32)$$

$$= \int_0^{\psi \max\left\{\begin{matrix}\rho(R^{q-1}x - R^{q-2}x), \rho(R^q x - R^{q-1}x), \\ \left[\frac{\rho(R^q x - R^{q-2}x) + \rho(R^{q-1}x - R^{q-2}x)}{2}\right]\end{matrix}\right\}} \xi(u)du$$

$$= \int_0^{\psi \max\{\rho(R^{q-1}x - R^{q-2}x), \rho(R^q x - R^{q-1}x)\}} \xi(u)du$$

If $\rho(R^q x - R^{q-1}x) > \rho(R^{q-1}x - R^{q-2}x) \geq 0$, then

$$m(R^{q-1}x - R^{q-2}x) = \rho(R^q x - R^{q-1}x)$$

$$\int_0^{\psi(\rho(R^q x - R^{q-1}x))} \xi(u)du \leq \int_0^{\psi(m(R^{q-1}x - R^{q-2}x))-\phi(m(R^{q-1}x - R^{q-2}x))} \xi(u)du$$

$$\leq \int_0^{\psi(m(R^q x - R^{q-2}x))-\phi(m(R^q x - R^{q-2}x))} \xi(u)du \qquad \ldots(33)$$

$$< \int_0^{\psi\left(\rho\left(R^q x - R^{q-1}x\right)\right)} \xi(u)du$$

Which is a contradiction, and hence

$\rho(R^q x - R^{q-1}x) \leq m(R^{q-1}x - R^{q-2}x)$

$$= \rho(R^{q-1}x - R^{q-2}x) \qquad \ldots(34)$$

So, that the sequence $\{\rho(R^q x - R^{q-1}x)\}$ is monotone decreasing and bounded below. Hence there exists $k \geq 0$ such that $\lim_{n\to\infty} \rho(R^q x - R^{q-1}x) = k \qquad \ldots(35)$

If $k > 0$, at $n \to \infty$ the inequality (30), we get

$\psi(k) \leq \psi(k) - \phi(k)$

$$< \psi(k) \qquad \ldots(36)$$

Which is a contradiction, and thus $k = 0$, so we have

$$\lim_{n\to\infty} \rho(R^q x - R^{q-1}x) = 0 \qquad \ldots(37)$$

Next we prove that the sequence $\{R^q(x)\}_{q\in N}$ is not a ρ-cauchy and there exist $\epsilon > 0$ and sequence$\{p_n\} > \{q_n\} \geq n$ such that

$\rho(R^{p_n}x - R^{q_n}x) \geq \epsilon, \rho\big(2(R^{p_n-1}x - R^{q_n}x)\big) < \epsilon \qquad \ldots(38)$

Since,

$$\int_0^{\psi(\rho(R^{p_n}x - R^{q_n}x))} \xi(u)du \leq \int_0^{\psi\left(m(R^{p_n-1}x,\ R^{q_n-1}x)\right) - \phi\left(m(R^{p_n-1}x,\ R^{q_n-1}x)\right)} \xi(u)du \quad (39)$$

$$\leq \int_0^{\psi\left(m(R^{p_n-1}x,\ R^{q_n-1}x)\right)} \xi(u)du$$

Which implies that

$$\rho(R^{p_n}x - R^{q_n}x) \leq m(R^{p_n-1}x,\ R^{q_n-1}x) \ldots(40)$$

Also, $$m(R^{p_n-1}x,\ R^{q_n-1}x) = max\begin{Bmatrix} \rho(R^{p_n-1}x,\ R^{q_n-1}x), \rho(R^{p_n}x,\ R^{p_n-1}x), \rho(R^{q_n}x,\ R^{q_n-1}x), \\ \left\{\rho\left(\frac{1}{2}(R^{p_n}x,\ R^{q_n-1}x)\right) + \rho\left(\frac{1}{2}(R^{p_n-1}x,\ R^{q_n}x)\right)\frac{1}{2}\right\} \end{Bmatrix} \ldots(41)$$

$$\rho(R^{p_n-1}x,\ R^{q_n-1}x) = \rho(R^{p_n-1}x - R^{q_n}x + R^{q_n} - R^{q_n-1}x)$$

$$\leq \rho(2(R^{p_n-1}x - R^{q_n})) + \rho(2(R^{q_n}x - R^{q_n-1}x))$$

$$< \epsilon + \rho(2(R^{q_n}x - R^{q_n-1}x))$$

For the last term in $m(R^{p_n-1}x,\ R^{q_n-1}x)$, by proposition (2.1), we have

$$\int_0^{\psi\left\{\rho\left(\frac{1}{2}(R^{p_n}x - R^{q_n-1}x)\right) + \rho\left(\frac{1}{2}(R^{p_n-1}x - R^{q_n}x)\right)\frac{1}{2}\right\}} \xi(u)du$$

$$= \int_0^{\psi\left\{\rho\left(\frac{1}{2}(R^{p_n-1}x - R^{q_n}x)\right) + \rho\left(\frac{1}{2}(R^{p_n}x - R^{p_n-1}x + R^{p_n-1}x - R^{q_n}x) + \frac{1}{2}(R^{q_n}x - R^{q_n-1}x)\right)\frac{1}{2}\right\}} \xi(u)du$$

$$\leq \int_0^{\psi\left\{\rho\left(\frac{1}{2}(R^{p_n-1}x - R^{q_n}x)\right) + \rho\left(\frac{1}{2}(R^{p_n}x - R^{p_n-1}x + R^{p_n-1}x - R^{q_n}x) + \frac{1}{2}(R^{q_n}x - R^{q_n-1}x)\right)\frac{1}{2}\right\}} \xi(u)du \quad \ldots(42)$$

$$\leq \int_0^{\psi\left\{\rho\left(\frac{1}{2}(R^{p_n-1}x - R^{q_n}x)\right) + \rho\left(2(R^{p_n}x - R^{p_n-1}x)\right) + \rho\left(2(R^{p_n-1}x - R^{q_n}x)\right) + \rho\left((R^{q_n}x - R^{q_n-1}x)\right)\frac{1}{2}\right\}} \xi(u)du$$

$$< \epsilon + \int_0^{\psi\left\{\left[\rho(R^{q_n}x - R^{q_n-1}x) + \rho(R^{p_n}x - R^{p_n-1}x)\right]\frac{1}{2}\right\}} \xi(u)du$$

It follows from (41) and (42) we have

$$m(R^{p_n-1}x,\ R^{q_n-1}x) = max\begin{Bmatrix} \rho(R^{p_n-1}x - R^{q_n-1}x), \rho(R^{p_n}x - R^{p_n-1}x), \rho(R^{q_n}x - R^{q_n-1}x), \\ \left\{\rho\left(\frac{1}{2}(R^{p_n}x - R^{q_n-1}x)\right) + \rho\left(\frac{1}{2}(R^{p_n-1}x - R^{q_n}x)\right)\frac{1}{2}\right\} \end{Bmatrix} \ldots(43)$$

$$< max\begin{Bmatrix} \epsilon + \rho\big(2(R^{q_n}x - R^{q_n-1}x)\big), \rho(R^{p_n}x - R^{p_n-1}x), \rho(R^{q_n}x - R^{q_n-1}x), \quad \epsilon, \\ \left[\rho(R^{q_n}x - R^{q_n-1}x) + \rho(R^{p_n}x - R^{p_n-1}x)\right]\frac{1}{2} \end{Bmatrix}$$

By (37), (38), (40) and (43) and the ∇_2- condition of ρ, we have

$$\lim_{n\to\infty}(R^{p_n}x - R^{q_n}x) = \lim_{n\to\infty} m(R^{p_n-1}x - R^{q_n-1}x) = \epsilon \qquad \ldots(44)$$

Taking $n \to \infty$ in (39) by (44) and the continuity of ψ, we get

$$\psi(\epsilon) \leq \psi(\epsilon) - \phi(\epsilon) < \psi(\epsilon) \qquad \ldots(45)$$

which is a contraction. Hence the sequence $\{R^q(x)\}_{q\in N}$ is a ρ-cauchy. Since X_ρ is a ρ-Complete and there exist a point $v \in X_\rho$ such that $\rho(R^{q_n}x - v) \to 0$ as $q \to \infty$.

Next we have to prove that v is a unique fixed point of R, for this, Let us assume that $Rv \neq v,\ then\ \rho(v - Rv) > 0.$

$\int_0^{\psi(\rho(R^q x - Rv))} \xi(u)du \leq \int_0^{\psi\left(m(R^{q-1}x,\ v)\right) - \phi\left(m(R^{q-1}x,\ v)\right)} \xi(u)du$ (46)

$$m(R^{q-1}x,\ v) = max\begin{Bmatrix} \rho(R^{q-1}x - v), \rho(R^{q-1}x - R^q x), \rho(v - Rv), \\ \left\{\rho\left(\frac{1}{2}(R^{q-1}x,\ Rv)\right) + \rho\left(\frac{1}{2}(v - R^q x)\right)\frac{1}{2}\right\} \end{Bmatrix} \quad \ldots(47)$$

$$= max\left\{0, 0, \rho(v - Rv), \frac{\rho\left(\left(\frac{1}{2}\right)(v - Rv)\right)}{2}\right\}$$

$= \rho(v - Rv)\ \ as\ \ q \to \infty.$

Now as $q \to \infty$ in (46) by using (47) we get

$$\int_0^{\psi(\rho(v-Rv))} \xi(u)du \leq \int_0^{\psi(\rho(v-Rv)) - \phi(\rho(v-Rv))} \xi(u)du$$

$$< \int_0^{\psi(\rho(v-Rv))} \xi(u)du \quad ...(48)$$

Which is a contradiction. Hence $\rho(v - Rv) = 0\ and\ Rv = v$, If there exist point $w \in X_\rho$ such that $Rw = w\ and\ \ v \neq w$, then using an argument, we get

$$\int_0^{\psi(\rho(v-w))} \xi(u)du = \int_0^{\psi(\rho(Rv-Rw))} \xi(u)du$$

$$\leq \int_0^{\psi(m(v,w)) - \phi(m(v,w))} \xi(u)du$$

$$\leq \int_0^{\psi(\rho(v-w)) - \phi(\rho(v-w))} \xi(u)du \ ...(49)$$

$$< \int_0^{\psi(\rho(v-w))} \xi(u)du$$

Which is a contradiction, Hence $v = w$.

Corollary: 2.6. Let X_ρ be a ρ-Complete modular space where ρ satisfy the Δ_2-condition and Let $R : X_\rho \to X_\rho$ be a mapping satisfying the inequality

$$\int_0^{\psi\left(\rho(R_x - R_y)\right)} \xi(u)du \leq \int_0^{\psi(m(x,y)) - \phi(m(x,y))} \xi(u)du \quad \ldots(50)$$

For all x, y X_ρ, where

$$m(x, y) = max\begin{Bmatrix} \rho(x - y), \rho(x - Rx), \rho(y - Ry), \\ \left\{\rho\left(\frac{1}{2}(x - Ry)\right) + \rho\left(\frac{1}{2}(y - Rx)\right)\frac{1}{2}\right\} \end{Bmatrix} \quad \ldots(29)$$

and $\psi : [0, \infty) \to [0, \infty)$ is continuous and monotonic non decreasing function with $\phi(u) = 0$ Iff $u = 0$, Then $\xi : R_+ \to R_+$ is a Lebesgueintegrable mapping is summable on each compact subset of$[0, \infty)$ nonnegative and $\int_0^{\epsilon} \xi(u)du = 0,\ \forall\ \ \varepsilon > 0.\ \psi$

Proof: Taking $\psi(u) = u$, we obtain the corollary.

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